ORNL SNS Accelerator Physics Internal Memo

Subject: Synchronous Particle Scoping Studies for an SNS SC Linac

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INTRODUCTION

A simple computer code for the acceleration of the synchronous particle has been written to perform general scoping studies for a possible SNS superconducting linac. The issue of the optimum number of cells per cavity and also the issue of designing the linac for constant energy gain per cavity or constant electric field per cavity are studied. The model for the calculation is described in the next section, followed by the scoping studies. A benchmark with a similar LANL calculation has also been completed.

CALCULATION

General Equations

The acceleration of the synchronous particle is calculated by integrating the following coupled equations across the individual accelerating cells,

$$\Delta E_i = E_{0-i} L_{cell} T_i \cos f_i$$

$$\Delta f_i = -p\left(1 - \frac{b_{i+1}}{b_g}\right) + \Delta f_{end}$$

where,

f_i = the phase in the center of cell i
L_{cell} = the cell length, given by $b_g l/2$
E_{0-i} = the average axial electric field in cell i
T_i = the transit time factor for cell i
b_i = the particle relativistic velocity factor in cell i
b_g = the design beta value of the cavity
Df_{end} = correction term for the cavity end cells.

Transit Time

The ideal transit time factor for an individual cell is taken from Ref. 1 and as is given by the equation,

$$T_0 = \frac{p}{4}\left[\frac{\sin\left(\frac{p}{2}\left(\frac{b_g}{b}+1\right)\right)}{\frac{p}{2}\left(\frac{b_g}{b}+1\right)}+\frac{\sin\left(\frac{p}{2}\left(\frac{b_g}{b}-1\right)\right)}{\frac{p}{2}\left(\frac{b_g}{b}-1\right)}\right].$$

An average transit time factor for a cavity is defined by the equation,

$$T_{cav} = \frac{\sum_{i=1}^{N} T_i \cos(f_i)}{\cos(< f >)},$$

where,

$<f\ > = (f_{N/2-1}+f_{N/2})/2$ is the desired particle phase at the center of the cavity,
N = the number of cells per cavity, and
T_i = the individual transit time factor for cell i, which is obtained from the ideal transit time factor using the correction described below.

<u>Correction Terms</u>

The ideal cavity transit time factors are multiplied by correction factors to reproduce the more accurate transit time factors, electric fields and cavity electrical centers calculated by Superfish. In particular, the transit time factors are corrected as follows:

$$T = T_0 \times \begin{cases} 0.99 \text{ for interior cells} \\ 0.75 + 0.556(b_g - b_i) + (b_i - b_g)(0.5 - 1.333(b_g - b_i)) \text{ for end cells} \end{cases}$$

Also, correction factors derived from Superfish runs are included to account for the fact that the end cell electric fields are typically larger than those of the interior cells. The electric fields in the end cells for 8 cell cavities are scaled as

$$E_{0\text{-}end} = E_{0\text{-}int}\,(1.3328 - 0.613(b_g - 0.61))$$

while, for 6 cell cavities, the electric fields are increased by

$$E_{0\text{-}end} = E_{0\text{-}int}\,(1.32 - 0.8(b_g - 0.61)).$$

The average longitudinal electric field in a cavity is given by the equation,

$$< E_0 >= \sum_{i=1}^{N} \frac{E_{0-i}}{N}.$$

Finally, the phase advance of a particle accelerating from an end to interior cell or vice versa is modified by the fact that the distance between the end cell electrical center and adjacent interior cell electrical center is slightly larger than $b_g l/2$. Hence, it takes slightly longer for the synchronous particle to pass from end to interior electrical center than to pass from interior to interior cell. Accordingly, the end cell phase advance is increased by the amount

$$\Delta f_{end} = p\left(0.088 - 0.18(b_g - 0.61)\right) ,$$

as derived from Superfish calculations at different design betas. These end cell effects combine to decease the total required electric field. The above ad hoc end cell correction factors were derived for b_g = 0.61 and b_g = 0.76 Superfish calculations only, and should not be considered accurate for other b values. These correction values were derived to serve as general guides for this initial optimization study.

Integration Schemes

Integration through the cells of a particular cavity is accomplished with a function evaluator. The function evaluator is called by an equation solver, which can be configured in several ways. For cases with constant cavity $<E_0>$ and fixed $< f >$, $E_{0\text{-}int}$ is prescribed and the initial phase, f_1, is iterated until the desired $< f >$ is obtained. For cases with prescribed $<f>$ and prescribed energy gain per cavity, both f_1 and $E_{0\text{-}int}$ are simultaneously iterated to obtain the desired energy gain per cavity and $<f>$. This procedure is applied to the entire set of specified cavities.

Linac Architecture

Two cavity families, b_g = 0.61 and b_g = 0.76, where every cavity in a family has the same design b_g, were assumed for all cases. Both Wangler and Pigani use these two values of b_g for an SNS SC linac. In order to determine the actual real-estate length of the linac additional hardware length requirements must be specified. In this study, the following lengths were adopted: 17-cm lengths on either side of each cavity for cavity ends; 5-cm lengths on both sides of each cavity for bellows; 30-cm lengths for warm to cold transitions at the end of each cryomodule; and 175-cm warm lengths between each cryomodule for quadrupoles, correctors, vacuum equipment, and diagnostics. The number of cavities per cryomodule is listed below.

	Number of cavities / cryomodule	
	β = 0.61 cavity	β= 0.76 cavity
6 cells/cavity	3	4
8 cells/cavity	2	3

Peak Cavity Surface Fields

The ratio of the peak surface electric field, E_{peak}, to $E_{0\text{-}int}$ and the peak surface magnetic field, B_{peak}, to $E_{0\text{-}int}$ are input for each cavity family. These ratios were obtained from design calculations with Superfish for a particular cavity that was selected as a reasonable compromise between many available cavity designs with 5-cm radius apertures. The peak surface fields provide a fundamental limitation on the acceleration electric field. Unless stated otherwise the following ratios were used. The peak surface electric field was limited to 33 MV/m which corresponds to peak magnetic field of less than about 70 mT or 700 Gauss.

Cavity:	$E_{peak}/E_{0\text{-}int}$	B_{peak}/E_{peak} (mT/MV/m)
$b_g = 0.61$	2.33	1.99
$b_g = 0.76$	1.82	2.12

Beam and Coupler Power Requirements

KEK has built and tested input power couplers with peak power capabilities of 380 kW. For this study we assume conservatively a maximum coupler power to a cavity of 350 kW. The average beam current for the 2 MW SNS beam during a macropulse is 36 mA, that is the product of the peak macropulse current of 56 mA and the 65% beam-on chopper duty factor. Only the beam power is considered, no losses are accounted for.

Optimizer

The model described above can be called from a genetic optimization package to facilitate constrained optimization. The figure-of-merit, and constraint choice are quite flexible. Some variables, constraints, and figures of merit for problem formulation choices are listed below:

- Minimize the total linac length
- Vary the number of cavities per family
- Vary $E_{0\text{-}int}$ for each family
- Vary b_g of each family
- Constrain E_{peak} below some prescribed limit
- Constrain the beam power for a cavity to a prescribed limit
- Constrain the final linac energy to a prescribed value.

SCOPING STUDIES

In this section, we derive a number of optimized linac architectures using the above model. The results are summarized in Table 1. In order to benchmark the calculation, we first show a comparison with results from the LANL calculation of Ref. 2. Following this benchmark we examine the impact of choosing 6 or 8 cells per cavity, and the impact of constant power per cavity or constant electric field per cavity. For an SNS superconducting linac the starting energy is taken to be 194.3 MeV and the final energy is taken to be 1001.5 MeV. Two cavity families are assumed: $b_g = 0.61$ and $b_g = 0.76$.

LANL Benchmark

As a benchmark of our model, we reproduce the results of T. Wangler et. al. [2]. For this case there is no optimization and the parameters are set to those described in Ref. 2. In particular, these parameters are:

- Two families of 8 cell cavities. The first family consists of 30 cavities at b_g=0.61 and the second family contains 72 cavities at b_g=0.76.
- The phases <f> in the first family are ramped linearly from –25 degrees to –39 degrees, and the phases in the second family are all fixed at –26.5 degrees.
- The energy gain/cavity in each cell is fixed at 5.37 MeV for the low b family, and at 8.95 MeV for the high b family.
- The required accelerating gradient in each cavity is calculated.
- $E_{peak}/ E_{0\text{-}int}$ = 2.91 and 2.20 for the low and high beta cavities, respectively.
- B_{peak}/ E_{peak} = 1.7 mT/MV/m for both the low and high beta cavities.

The results from this calculation are listed in the Table 2. Overall, the parameters are very close to those listed in Ref. 2. A comparison of the calculated transit times and required cavity electric fields to produce the prescribed energy gain per cavity is shown in Figure 1. For comparison to Ref. 2, the quantity E_a shown in Fig. 1 is defined as $E_a = \langle E_0 \rangle \langle T \rangle_{\max}$, where $\langle T \rangle_{\max}$ is the maximum cavity transit time factor for the entire cavity family. The agreement between the two calculations is good and establishes a benchmark for the present calculation. The length of the linac is 242 meters with 102 cavities and 39 cryomodules.

<u>Fixed Maximum Peak Surface Electric Field</u>

In this section the case with a fixed maximum peak surface electric field is examined for six and eight cells per cavity. Because the ratio of peak surface field to the average electric field varies with beta, this assumption results in a fixed field for each cavity family. The energy gain and power requirements per cavity are then calculated. The optimization formulation is listed below:

- Use 2 families of 6 cell or 8 cell cavities
- Prescribe the average phase
  - The first family has its phases ramped from –25 to –39 degrees
  - The second family has its phases fixed at –26.5 degrees
- Prescribe identical electric fields in every cavity in each family
- Calculate the energy gain for each cavity
- Constrain the peak surface field to be $\leq$ 33 MV/m
- Constrain the power to each cavity or coupler to be $\leq$ 350 kW
- Optimize by varying the number of cavities per family and the cavity field for each family in order to
- Minimize the total real estate linac length

The resulting parameters for this case are listed in Table 3 for the 6 cell cavity case and in Table 4 for the 8 cell cavity case. The 6 cell cavity linac is ~ 25% shorter than the 8 cell cavity linac. The 8 cell cavity linac is much more limited by the 350-kW coupler-power constraint than is the 6 cell cavity linac. As a result of this power limit, the peak surface field for the β= 0.76 cavity family is well below the limit of 33 MV/m and the overall linac length is longer. The resulting phase for each cell, cavity average transit time factor, cavity electric field, and cavity beam power are shown in Figs. 2 and 3 for the 6 cell and 8 cell cases. The cavity transition time factor and cavity power curves are less peaked for the 6 cell case, indicating a more efficient use of the cavities. This increased efficiency results from the decreased spread in phase angles for the six cell case. In particular, Fig. 3a shows a maximum phase spread of about 145 degrees across the last cavitiy for the 8 cell case, whereas Fig. 2a shows about 100 degree phase spread for the 6 cell case.

<u>Fixed Energy Gain or Power Per Cavity and Coupler</u>

In this section the case with constant energy gain in each cavity in a family is examined. The energy gain per cavity is maintained at a constant value for each cavity in a family by varying the cavity electric field. The energy gain and power requirements for each cavity are calculated. The optimization formulation is listed below:

- Use 2 families of 6 cell or 8 cell cavities
- Prescribe the average phase
    - The first family has its phases ramped from –25 to –39 degrees
    - The second family has its phases fixed at –26.5 degrees
- Prescribe identical energy gain in every cavity in each family
- Calculate the electric field for each cavity
- Limit the peak field to be ≤ 33 MV/m
- Limit the power to each cavity or coupler to be ≤ 350 kW
- Optimize by varying the number of cavities and the energy gain in each family to
- Minimize the total real estate linac length

Results for the 6 cell cavity case are shown in Table 4 and the 8 cell cavity results are shown in Table 5. The total real estate linac length in the 6 cell cavity case is ~ 15% shorter than in the 8 cell cavity case. Some cavity parameters are shown in Figs. 4 and 5 for these cases.

The resulting parameters are listed in Table 5 for the 6 cell cavity case and Table 6 for the 8 cell cavity case. The 6 cell cavity linac is ~ 15% shorter than the 8 cell cavity linac case. The resulting phase for each cell, cavity average transit time factor, cavity electric fields, and cavity beam power are shown in Figs. 4 and 5 for the 6 cell and 8 cell cases. As with the constant electric field case, using 6 cells per cavity leads to smaller variations in the transit time factor across a family, and hence to smaller variations in the electric field. The 8 cell cavity linac of Table 6 is very similar to the LANL benchmark linac of Table 1; however, it is two cryomodules shorter because of a more favorable peak to average electric field ratio for the cavities.

SUMMARY

Table 1 summarizes some of the more important parameters for the four optimizations listed in Tables 3 to 6. The specific values of these parameters depend on the geometric assumptions for the individual cells. Regardless, there are some important tradeoffs between six cell and eight cell cavities and also between constant energy gain per cavity and constant electric field in each cavity. Some of the more important differences are:

- The quadrupole periods for the medium and low beta families are about 15% and 7% shorter, respectively, for the 8 cell cavity case than for the 6 cell cavity case. This results in slightly stronger transverse focusing and reduced transverse emittance growth in the 8 cell cavity case.

- The 6 cell cavity linacs are shorter than the corresponding 8 cell cavity linacs, even though the filling factors are slightly larger for the 8 cell linacs. This reduction in filling factor is more than offset by the increased transit time factors for the six cell case.

- The 6 cell constant gradient linac is shorter than the 6 cell constant energy gain linac because the latter is limited by the gradient of the end cavities, where the transit time factor is small.

REFERENCES

1. T. Wangler, “Principles of RF Linear Accelerators”, J. Wiley & Sons, 1998.

2. T. Wangler, private communication, August 1999.

Table 1. Summary of parameters for an optimized 805-MHz SC linac from 194.3 to 1001.5 MeV.

Constant gradient or beam power	Gradient		Gradient		Power		Power	
Cells per cavity	6		8		6		8	
Total real estate length [m]	191.2		243.1		206.8		234.7	
Total Cavity Length [m]	79.5		108.3		84.3		102.0	
Energy gain per real estate length [m]	4.22		3.32		3.90		3.44	
Beta	0.61	0.76	0.61	0.76	0.61	0.76	0.61	0.76
Active cavity length [m]	0.681	0.849	0.908	1.132	0.681	0.849	0.908	1.132
$E_{peak} / E_{0\text{-int}}$	2.33	1.82	2.33	1.82	2.33	1.82	2.32	1.82
B_{peak} / E_{peak} [mT/MV/m]	1.99	2.12	1.99	2.12	1.99	2.12	1.99	2.12
E_{peak} [MV/m]	32.6	29.1	32.6	21.7	32.2	32.4	30.4	318
B_{peak} [mT]	64.9	61.4	64.9	45.9	64.0	68.6	60.5	67.4
Max $<E_o>$ MV/m]	15.5	17.1	15.2	12.6	15.3	19.0	14.1	18.5
Min $<E_o>$ [MV/m]	15.5	17.1	15.2	12.6	12.1	16.0	9.95	12.9
Max energy gain / cavity [MeV]	6.39	9.58	8.54	9.50	5.05	8.98	5.61	9.68
Min energy gain / cavity [MeV]	5.28	7.21	6.08	6.48	5.05	8.98	5.61	9.68
Max peak power per cavity [kW]	230	345	307	342	182	323	202	348
Min peak power per cavity [kW]	190	260	219	233	182	323	202	348
No. cryomodules	9	18	11	26	13	17	15	22
No. cavities / cryomodule	3	4	2	3	3	4	2	3
No. cavities	27	72	22	78	39	68	30	66
No. of cells / cavity	6	6	8	8	6	6	8	8
Total family length [m]	52.5	138.6	56.1	187.0	75.9	130.9	76.5	158.2
Period length (m)	5.84	7.70	5.10	7.19	5.84	7.70	5.10	7.19
Starting phase [deg]	-25	-26.5	-25	-26.5	-25	-26.5	-25	-26.5
Ending phase [deg]	-39	-26.5	-39	-26.5	-39	-26.5	-39	-26.5
Transition energy [MeV]	356.4	356.4	360.9	360.9	391.2	391.2	362.7	362.7
Fill factor	0.350	0.441	0.356	0.472	0.350	0.441	0.356	0.472
Average cavity transit time factor	0.688	0.698	0.665	0.663	0.684	0.707	0.670	0.661
Average cavity $<\cos f>$	0.798	0.824	0.752	0.757	0.787	0.832	0.759	0.768
Average cavity $<E_0>$	15.5	17.1	15.2	12.6	13.2	16.9	11.3	14.9

Table 2. LANL benchmark calculation, using ORNL fixed distances.

Warm space for quads [m]	1.6	
Warm to cold space [m]	0.297	
Cavity ends [m]	0.22	
Bellows space [m]	0.07	
Total real estate length [m]	249.09	
Total cavity length [m]	108.77	
Energy gain per real estate length [MeV/m]	3.2338	

CAVITY FAMILY	1	2
Beta	0.61	0.76
Frequency [MHz]	805	805
Lambda [m]	0.37241	0.37241
k [1/m]	27.658	22.199
Beta-lambda [m]	0.22717	0.28303
Active cavity length [m]	0.90869	1.1321
E_{peak} / E_0 interior cell	2.91	2.2
B_{peak} / E_0 interior cell [mT/MV/m]	4.947	3.74
B_{peak} / E_{peak} [mT/MV/m]	1.7	1.7
E_{peak} [MV/m]	34.937	37.317
B_{peak} [mT]	59.393	63.438
Max E_0 interior cell [MV/m]	12.006	16.962
Min E_0 interior cell [MV/m]	8.7894	11.211
Max $\langle E_0 \rangle$ [MV/m]	13.005	17.983
Min $\langle E_0 \rangle$ [MV/m]	9.5207	11.886
Average $\langle E_0 \rangle$	10.7	13.8
Max energy gain / cavity [MeV]	5.37	8.95
Min energy gain / cavity [MeV]	5.37	8.95
Max power / cavity (MW)	0.19332	0.3222
Average cavity transit time factor	0.674	0.659
Average cavity $\langle \cos f \rangle$	0.767	0.769
No. cryomodules	15	24
No. cavities / cryomodule	2	3
No. cavities	30	72
No. of cells / cavity	8	8
Period length [m]	5.1014	7.1904
Cryomodule length [m]	3.5014	5.5904
Fill factor	0.35625	0.47235
Total family length [m]	76.521	172.57
Starting phase [deg]	-25	-26.5
Ending phase [deg]	-38.5	-26.5
Energy in [MeV]	194.3	355.4
Energy out [MeV]	355.4	999.8

Table 3. Parameters for the minimum length, fixed E_0/cavity, 6 cells/cavity case.

Warm space for quads [m]	1.6
Warm to cold space [m]	0.297
Cavity ends [m]	0.22
Bellows space [m]	0.07
Total real estate length [m]	191.15
Total cavity length [m]	79.536
Energy gain per real estate length [MeV/m]	4.221

CAVITY FAMILY	1	2
Beta	0.61	0.76
Frequency [MHz]	805	805
Lambda [m]	0.37241	0.37241
k [1/m]	27.658	22.199
Beta-lambda [m]	0.22717	0.28303
Active cavity length [m]	0.68151	0.8491
E_{peak} / E_0 interior cell	2.33	1.82
B_{peak} / E_0 interior cell [mT/MV/m]	4.6367	3.8584
B_{peak} / E_{peak} [mT/MV/m]	1.99	2.12
E_{peak} [MV/m]	32.62	29.12
B_{peak} [mT]	64.914	61.734
Max E_0 interior cell [MV/m]	14	16
Min E_0 interior cell [MV/m]	14	16
Max $\langle E_0 \rangle$ [MV/m]	15.493	17.067
Min $\langle E_0 \rangle$ [MV/m]	15.493	17.067
Max energy gain / cavity [MeV]	6.3875	9.5759
Min energy gain / cavity [MeV]	5.2815	7.2053
Max power / cavity (MW)	0.22995	0.34473
Average cavity transit time factor	0.688	0.698
Average cavity $\langle \cos f \rangle$	0.798	0.824
No. cryomodules	9	18
No. cavities / cryomodule	3	4
No. cavities	27	72
No. of cells / cavity	6	6
Period length [m]	5.8385	7.7004
Cryomodule length [m]	4.2385	6.1004
Fill factor	0.35018	0.44107
Total family length [m]	52.547	138.61
Starting phase [deg]	-25	-26.5
Ending phase [deg]	-39	-26.5
Energy in [MeV]	194.31	356.36
Energy out [MeV]	356.36	1001.2

Table 4. Parameters for the minimum length, fixed E_0/cavity, 8 cells/cavity case.

Warm space for quads [m]	1.6	
Warm to cold space [m]	0.297	
Cavity ends [m]	0.22	
Bellows space [m]	0.07	
Total real estate length [m]	243.07	
Total cavity length [m]	108.3	
Energy gain per real estate length [MeV/m]	3.3174	
CAVITY FAMILY	1	2
Beta	0.61	0.76
Frequency [MHz]	805	805
Lambda [m]	0.37241	0.37241
k [1/m]	27.658	22.199
Beta-lambda [m]	0.22717	0.28303
Active cavity length [m]	0.90869	1.1321
E_{peak} / E_0 interior cell	2.33	1.82
B_{peak} / E_0 interior cell [mT/MV/m]	4.6367	3.8584
B_{peak} / E_{peak} [mT/MV/m]	1.99	2.12
E_{peak} [MV/m]	32.62	21.658
B_{peak} [mT]	64.914	45.915
Max E_0 interior cell [MV/m]	14	11.9
Min E_0 interior cell [MV/m]	14	11.9
Max $\langle E_0 \rangle$ [MV/m]	15.165	12.617
Min $\langle E_0 \rangle$ [MV/m]	15.165	12.617
Max energy gain / cavity [MeV]	8.5381	9.4996
Min energy gain / cavity [MeV]	6.0766	6.479
Max power / cavity (MW)	0.30737	0.34198
Average cavity transit time factor	0.665	0.653
Average cavity $\langle \cos f \rangle$	0.752	0.757
No. cryomodules	11	26
No. cavities / cryomodule	2	3
No. cavities	22	78
No. of cells / cavity	8	8
Period length [m]	5.1014	7.1904
Cryomodule length [m]	3.5014	5.5904
Fill factor	0.35625	0.47235
Total family length [m]	56.115	186.95
Starting phase [deg]	-25	-26.5
Ending phase [deg]	-39	-26.5
Energy in [MeV]	194.31	360.85
Energy out [MeV]	360.85	1000.7

Table 5. Parameters for the minimum length, fixed energy gain/cavity, 6 cells/cavity case.

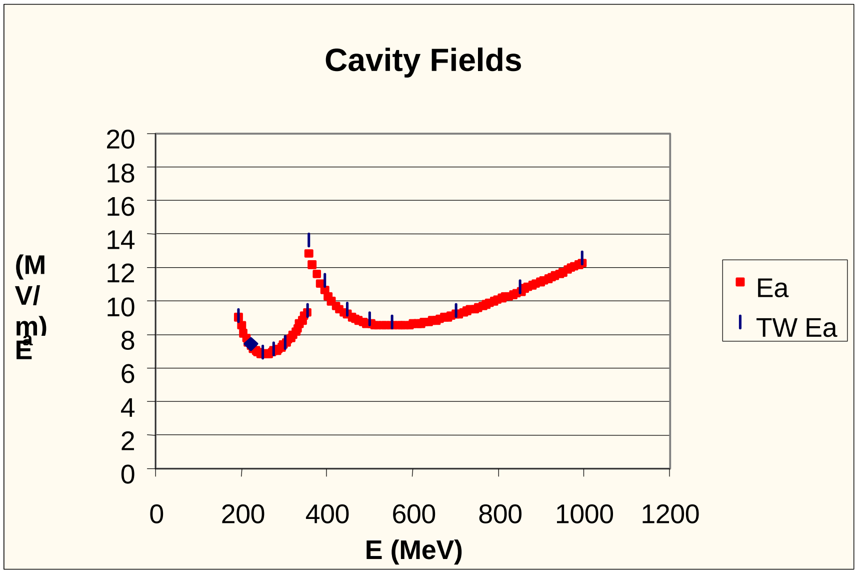
Warm space for quads [m]	1.6
Warm to cold space [m]	0.297
Cavity ends [m]	0.22
Bellows space [m]	0.07
Total real estate length [m]	206.81
Total cavity length [m]	84.318
Energy gain per real estate length [MeV/m]	3.9031

CAVITY FAMILY	1	2
Beta	0.61	0.76
Frequency [MHz]	805	805
Lambda [m]	0.37241	0.37241
k [1/m]	27.658	22.199
Beta-lambda [m]	0.22717	0.28303
Active cavity length [m]	0.68151	0.8491
E_{peak} / E_0 interior cell	2.33	1.82
B_{peak} / E_0 interior cell [mT/MV/m]	4.6367	3.8584
B_{peak} / E_{peak} [mT/MV/m]	1.99	2.12
E_{peak} [MV/m]	32.202	32.375
B_{peak} [mT]	64.081	68.634
Max E_0 interior cell [MV/m]	13.82	17.788
Min E_0 interior cell [MV/m]	10.932	14.995
Max $<E_0>$ [MV/m]	15.295	18.974
Min $<E_0>$ [MV/m]	12.098	15.994
Average $<E_0>$	13.1	16.9
Max energy gain / cavity [MeV]	5.0484	8.975
Min energy gain / cavity [MeV]	5.0484	8.975
Max power / cavity (MW)	0.18174	0.3231
Average cavity transit time factor	0.684	0.707
Average cavity $<\cos f>$	0.787	0.832
No. cryomodules	13	17
No. cavities / cryomodule	3	4
No. cavities	39	68
No. of cells / cavity	6	6
Period length [m]	5.8385	7.7004
Cryomodule length [m]	4.2385	6.1004
Fill factor	0.35018	0.44107
Total family length [m]	75.901	130.91
Starting phase [deg]	-25	-26.5
Ending phase [deg]	-39	-26.5
Energy in [MeV]	194.31	391.2
Energy out [MeV]	391.2	1001.5

Table 6. Parameters for the minimum length, fixed energy gain/cavity, 8 cells/cavity case.

Warm space for quads [m]	1.6
Warm to cold space [m]	0.297
Cavity ends [m]	0.22
Bellows space [m]	0.07
Total real estate length [m]	234.71
Total cavity length [m]	101.98
Energy gain per real estate length [MeV/m]	3.4391

CAVITY FAMILY	1	2
Beta	0.61	0.76
Frequency [MHz]	805	805
Lambda [m]	0.37241	0.37241
k [1/m]	27.658	22.199
Beta-lambda [m]	0.22717	0.28303
Active cavity length [m]	0.90869	1.1321
E_{peak} / E_0 interior cell	2.33	1.82
B_{peak} / E_0 interior cell [mT/MV/m]	4.6367	3.8584
B_{peak} / E_{peak} [mT/MV/m]	1.99	2.12
E_{peak} [MV/m]	30.383	31.805
B_{peak} [mT]	60.462	67.427
Max E_0 interior cell [MV/m]	13.04	17.475
Min E_0 interior cell [MV/m]	9.1886	12.125
Max $\langle E_0 \rangle$ [MV/m]	14.125	18.528
Min $\langle E_0 \rangle$ [MV/m]	9.9531	12.855
Average $\langle E_0 \rangle$	11.3	14.9
Max energy gain / cavity [MeV]	5.6136	9.6785
Min energy gain / cavity [MeV]	5.6135	9.6785
Max power / cavity (MW)	0.20209	0.34843
Average cavity transit time factor	0.670	0.661
Average cavity $\langle \cos f \rangle$	0.759	0.768
No. cryomodules	15	22
No. cavities / cryomodule	2	3
No. cavities	30	66
No. of cells / cavity	8	8
Period length [m]	5.1014	7.1904
Cryomodule length [m]	3.5014	5.5904
Fill factor	0.35625	0.47235
Total family length [m]	76.521	158.19
Starting phase [deg]	-25	-26.5
Ending phase [deg]	-39	-26.5
Energy in [MeV]	194.31	362.72
Energy out [MeV]	362.72	1001.5



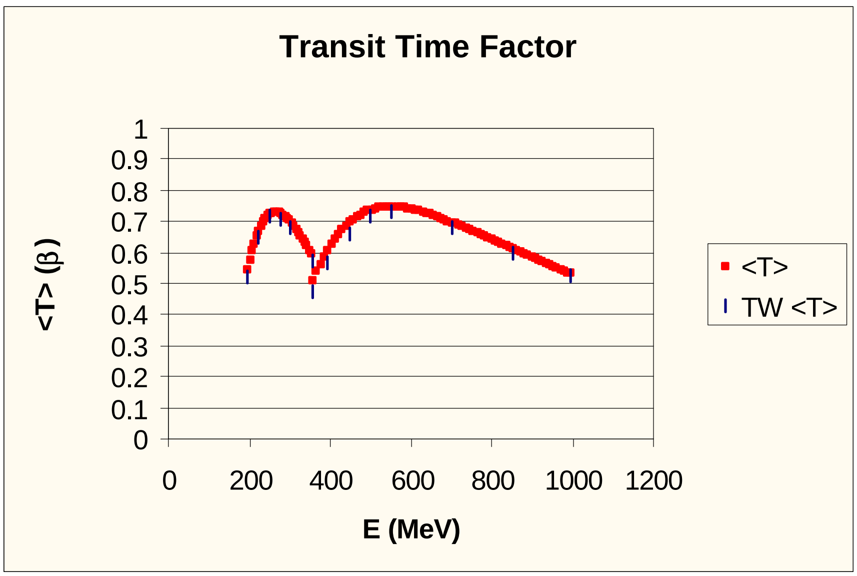


Fig. 1 Comparison of calculated cavity electric fields, in terms of E_a instead of E_0, and transit time factors with those of Ref. 2.

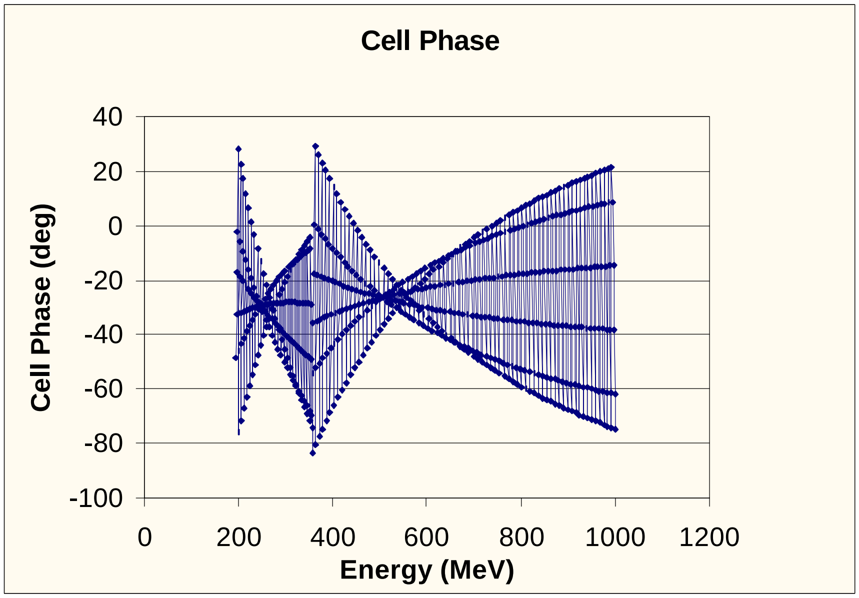


Fig 2a. Range of cell phases for each cavity for the constant E_0, 6 cells/cavity case.

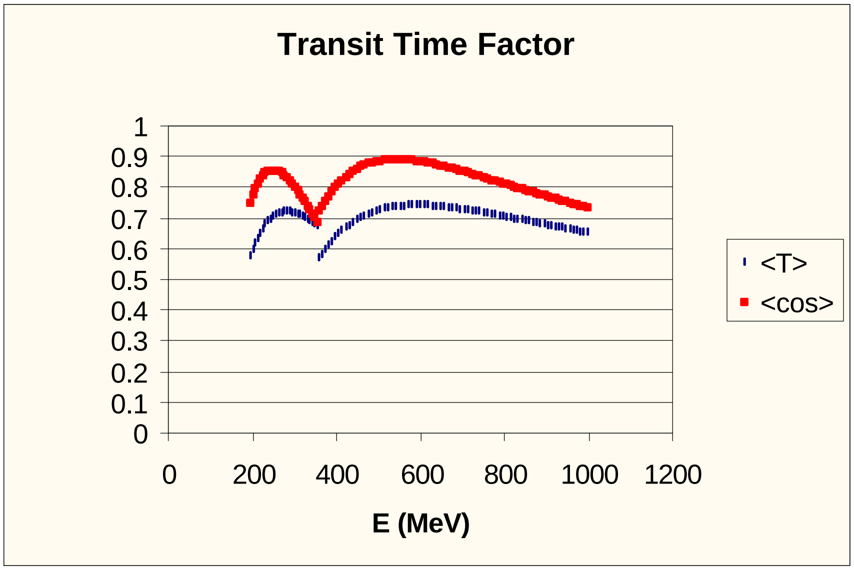


Fig 2b. Transit time factor, averaged over a cavity for the constant E_0, 6 cells/cavity case.

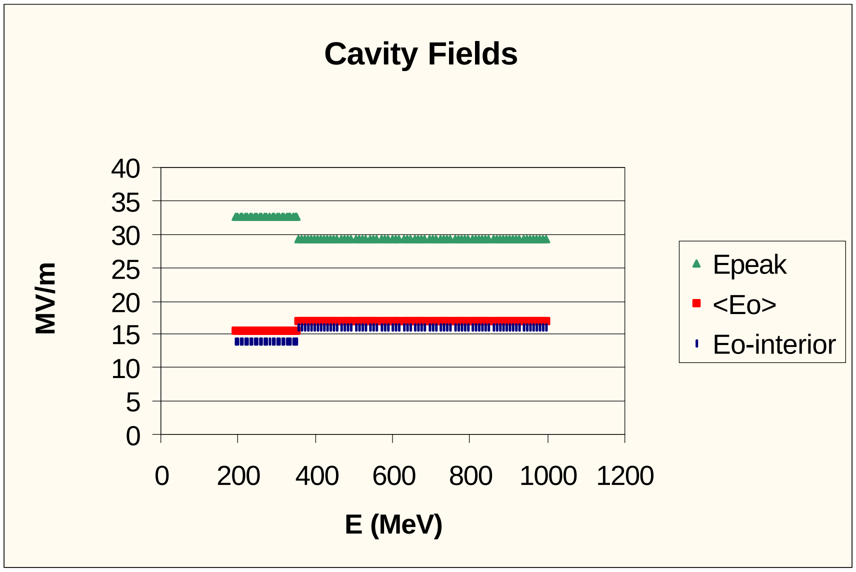


Fig. 2c. Cavity Fields for the constant E_0, 6 cells/cavity case.

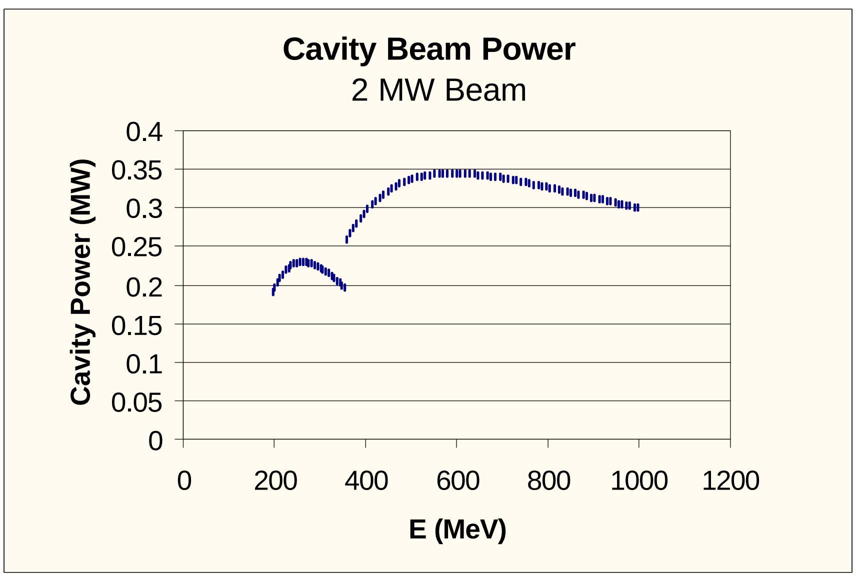


Fig. 2d. Cavity beam power for the constant E_0, 6 cells/cavity case.

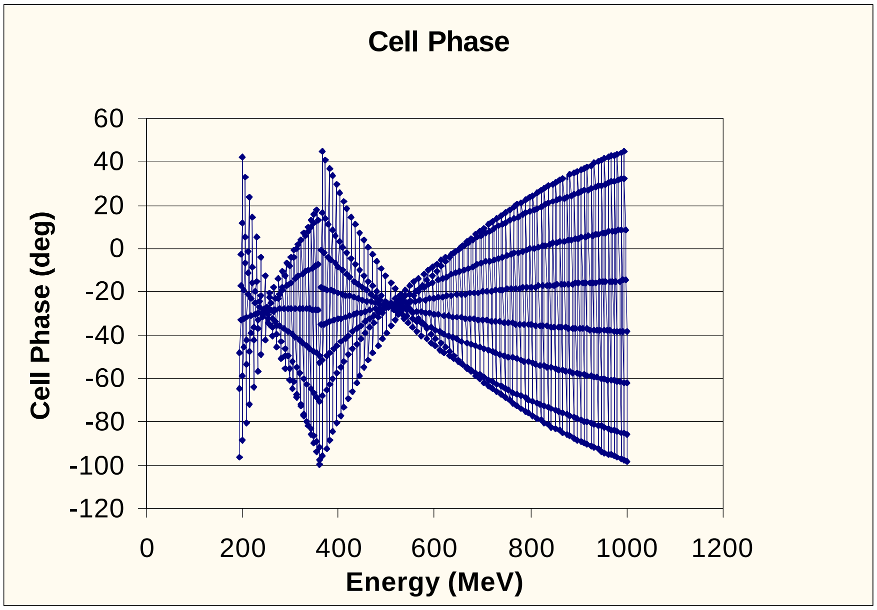


Fig. 3a. Range of cell phases for each cavity for the constant E_0, 8 cells/cavity case.

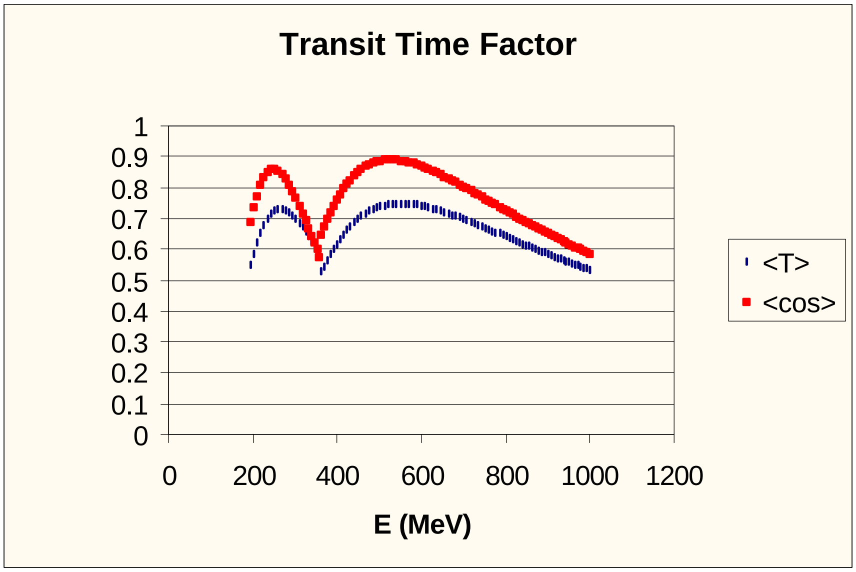


Fig. 3b. Transit time factor, averaged over a cavity for the constant E_0, 8 cells/cavity case.

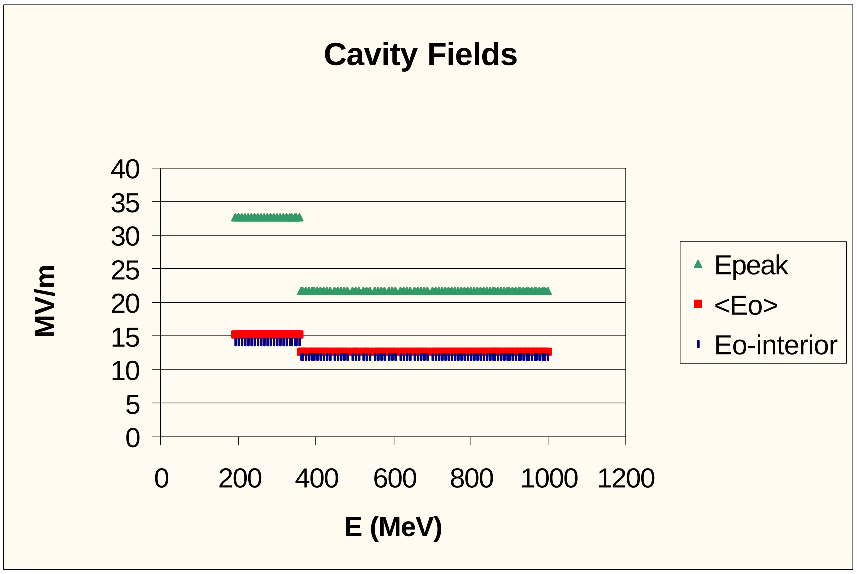


Fig. 3c. Cavity fields for the constant E_0, 8 cells/cavity case.

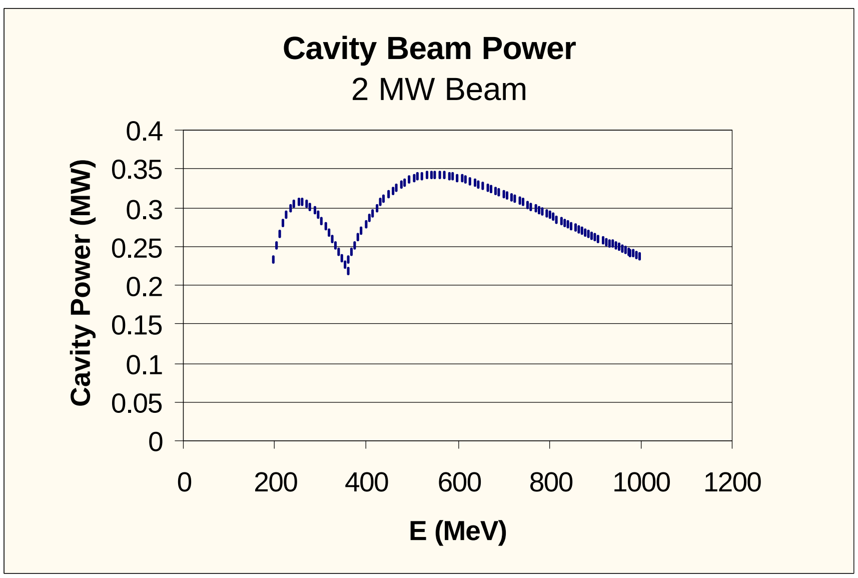


Fig.3d. Cavity beam power for the constant E_0, 8 cells / cavity case.

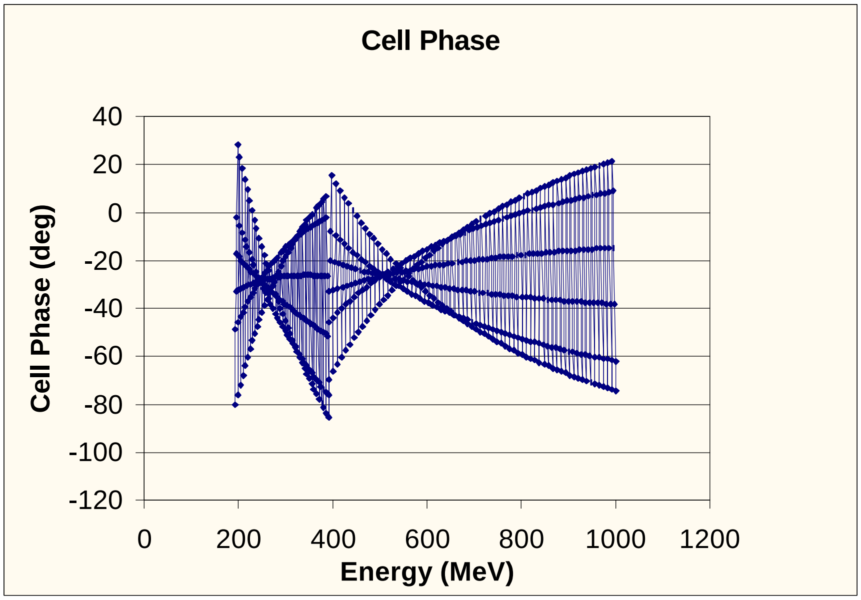


Fig. 4a Range of cell phases for each cavity for the constant energy gain/cavity, 6 cells/cavity case.

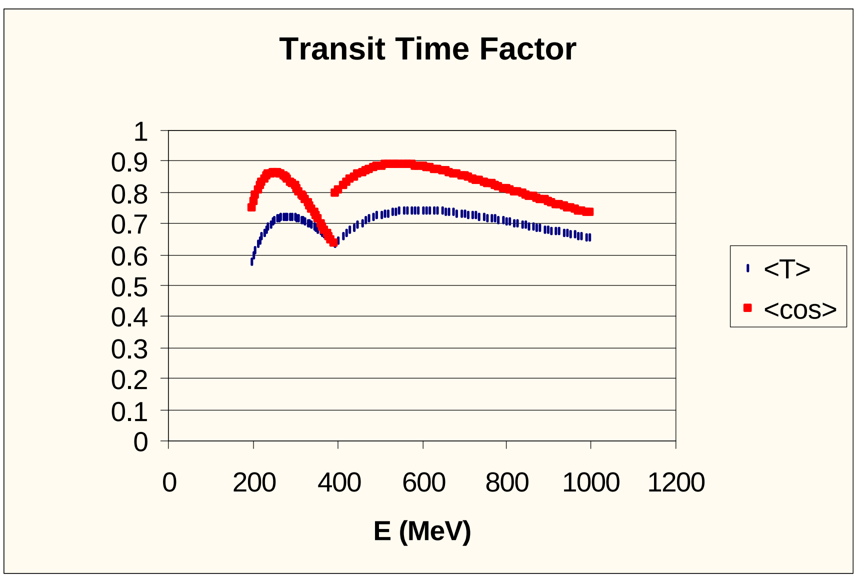


Fig. 4b. Transit time factor, averaged over a cavity for the constant energy gain/cavity, 6 cells/cavity case.

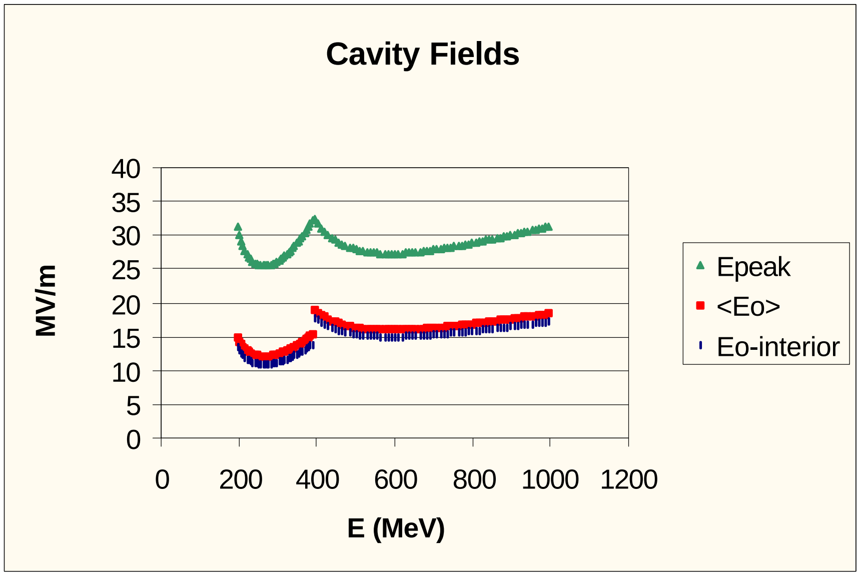


Fig. 4c. Cavity fields for the constant energy gain/cavity, 6 cells/cavity case.

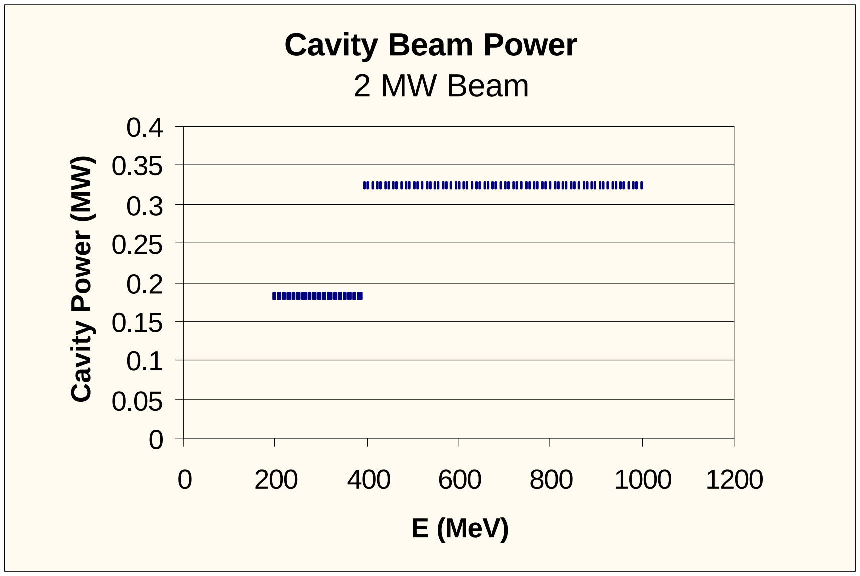


Fig.4d. Cavity beam power for the constant energy gain/cavity, 6 cells/cavity case.

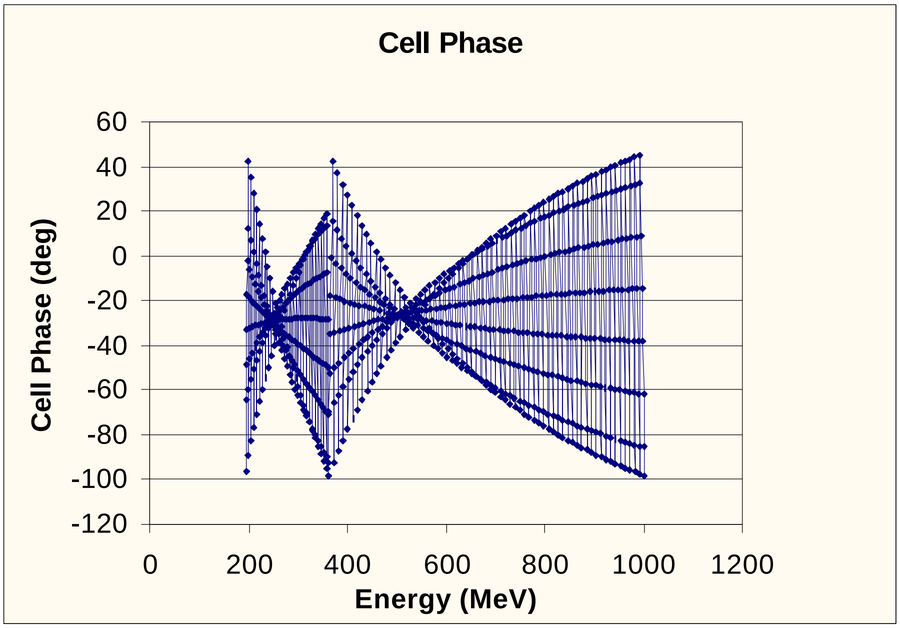


Fig. 5a. Range of cell phases for each cavity for the constant energy gain/cavity, 8 cells/cavity case.

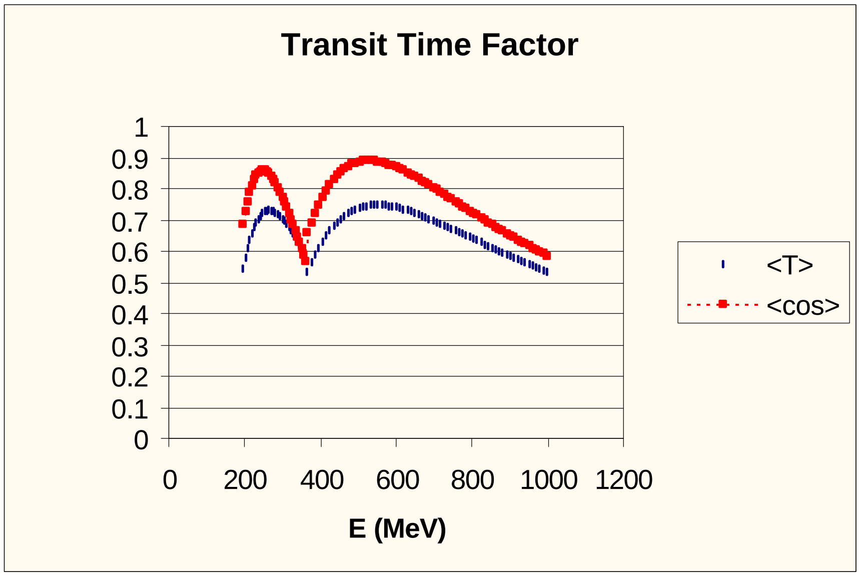


Fig. 5b. Transit time factor, averaged over a cavity for the constant energy gain/cavity, 8 cells/cavity case.

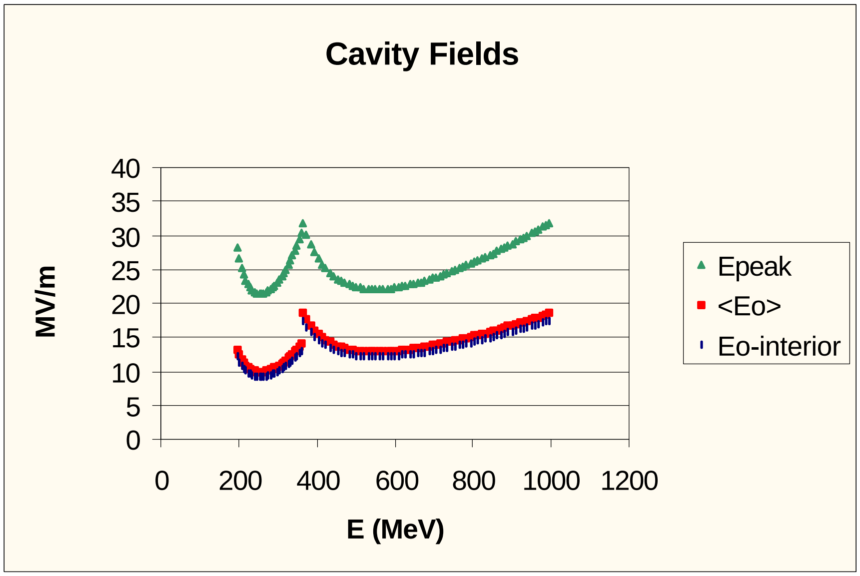


Fig. 5c. Cavity fields for the constant energy gain/cavity, 8 cells/cavity case.

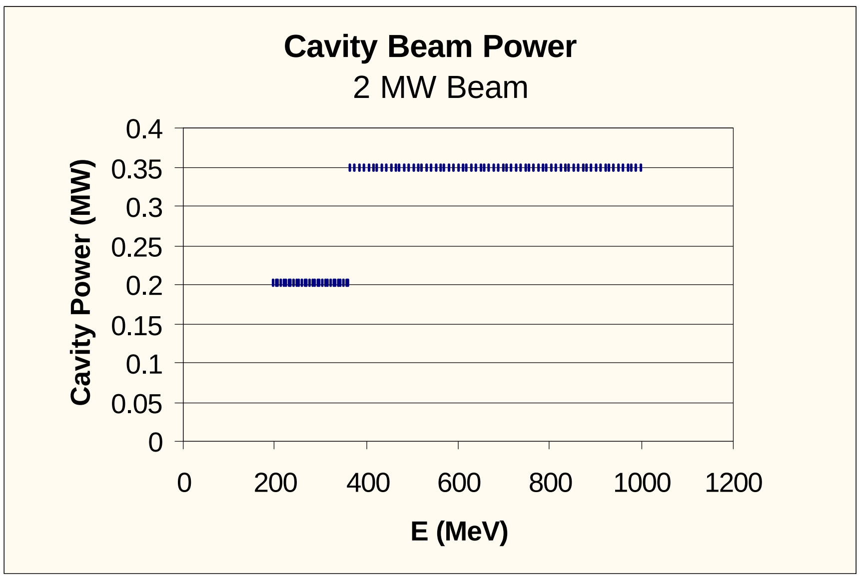


Fig. 5d. Cavity beam power for the constant energy gain/cavity, 8 cells/cavity case.